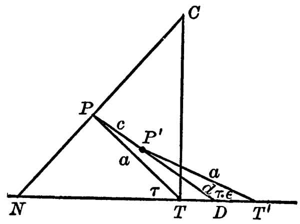
The pseudosphere is obtained by rotating the tractrix about its asymptote. That its total curvature is constant may be proved as follows: In any surface of revolution the principal radii of curvature are the radius of curvature of the meridian section and the length of the normal at the given point (Goursat-Hedrick, vol. 1, p. 505). As is well known the length of the tangent for the tractrix is a constant a; in fact, this is usually given as its definition. Let P and P' be two neighboring points on a tractrix, $PT = P'T' = a$ the tangents at these points, and τ and $\tau - \epsilon$, their *acute* inclinations to the axis. Produce $PP' = c$ to cut the axis in D and set $P'D = d$; then

$$\frac{c+d}{d} = \frac{a \sin \tau}{a \sin (\tau - \epsilon)} \quad \text{or} \quad \frac{c}{d} = \frac{\sin \tau - \sin (\tau - \epsilon)}{\sin (\tau - \epsilon)}.$$



Thus

$$\frac{c}{\epsilon} = d \frac{\cos\left(\tau - \frac{\epsilon}{2}\right)}{\sin (\tau - \epsilon)} \frac{\sin \frac{\epsilon}{2}}{\frac{\epsilon}{2}}.$$

As ϵ approaches zero, d approaches a and c/ϵ approaches the radius of curvature R. Hence $R = a \cot \tau$, and this shows that the line joining the center of curvature C with T is perpendicular to the axis. If PN is the normal at P, the right triangle CTN gives $CP \times PN = a^2$. (See Goursat-Hedrick, vol. 1, p. 441 for an analytical proof.) Thus the total curvature at every point of the pseudosphere is $-1/a^2$ and the torsion of an asymptotic line is $1/a$.

Also solved by J. B. REYNOLDS.

2782 [1919, 311]. Proposed by WARREN WEAVER, University of Wisconsin.

A great number, n, of jackstraws are jumbled up in such a way that any one is as likely to have one direction as another. Show that the probable number that make an angle lying between θ_1 and θ_2 as measured from any given direction is equal to $\dfrac{n(\cos \theta_1 - \cos \theta_2)}{2}$.

SOLUTION BY H. L. OLSON, University of Wisconsin.

Let us imagine the jackstraws all to have been translated in space so as to have *corresponding* ends coincident, and let us consider a sphere of unit radius with its center at this common end-point. The jackstraws satisfying the specified condition will then intersect the sphere in the points of a zone whose bases are circles with angular radii θ_1 and θ_2, respectively. Since the area of this zone is $2\pi(\cos \theta_1 - \cos \theta_2)$ (if $\theta_1 < \theta_2$) and the area of the entire sphere is 4π, the probable number of jackstraws making angles between θ_1 and θ_2 with a given line is $\dfrac{n(\cos \theta_1 - \cos \theta_2)}{2}$.

2789 [1919, 414]. Proposed by KURT LAVES, University of Chicago.

Given a quadrilateral $ABCD$ for which $|AC - BC| > |AD - BD|$[1] $(AC + BC < AD + BD)$ to construct, by means of the ruler and compass only, the pair of tangents from D to the hyperbola (ellipse) for which A and B are the foci and C a point on the hyperbola (ellipse).

SOLUTION BY A. PELLETIER, Montreal, Can.

Ellipse: D is outside the curve and the solution is always possible. With B as a center and a radius of length $AC + BC$ describe the circle x; with D as a center and DA as a radius, describe the circle y. The circles x and y intersect at M and N. The perpendiculars erected on AM, AN, at the mid-points of these lines are the required tangents, as is well known.

Hyperbola: The construction is the same as above, the radius of x being $|AC - BC|$.

[1] This expression was inadvertently omitted when published originally.